Application of granular jamming: robots!

Cornell (Amend and Lipson groups) in collaboration with Univ of Chicago (Jaeger group)

Lecture 2: (complex) fluid mechanics for physicists

S-RSI Physics Lectures:
Soft Condensed Matter Physics

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2012

Note: I have added links addressing questions and topics from lectures at:

http://conradlab.chee.uh.edu/srsi_links.html

Email me questions/comments/suggestions!

Soft condensed matter physics

- Lecture 1: statistical mechanics and phase transitions via colloids
- **Lecture 2: (complex) fluid mechanics for physicists**
- Lecture 3: physics of bacteria motility
- Lecture 4: viscoelasticity and cell mechanics
- Lecture 5: Dr. Conrad's work

Big question for today's lecture

How does the fluid mechanics of complex fluids differ from that of simple fluids?

Examples of complex fluids:

Petroleum

Food products

Personal care products

Ceramic precursors

Paints and coatings

Topic 1: shear thickening

Forces and pressures

A force causes an object to change velocity (either in magnitude or direction) or to deform (i.e. bend, stretch).

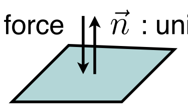
Newton's second law: $$\underbrace{\sum \vec{F}}_{\text{net force}} = \underbrace{m}_{\text{mass}}\underbrace{\vec{a}}_{\text{acceleration}} = \underbrace{\frac{d\vec{p}}{dt}}_{\text{change in linear momentum over time}}$$

$$\vec{p} = m\vec{v}$$

acceleration $\vec{a} = \dfrac{d\vec{v}}{dt}$

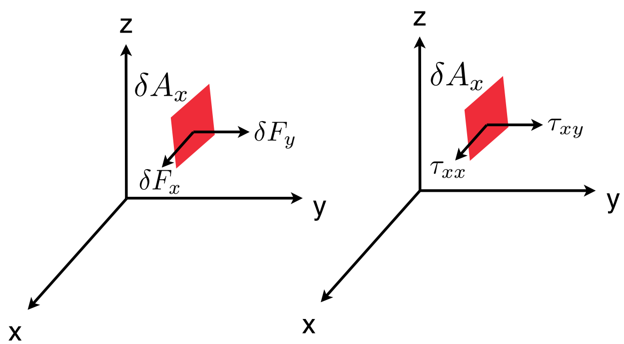
A pressure is a force/unit area applied perpendicular to an object.
Example: wind blowing on your hand.

direction of pressure force $\vec{n}$: unit vector normal to the surface



Stress

A stress is a force per unit area that is measured on an infinitely small area. Because forces have three directions and surfaces have three orientations, there are nine components of stress.



Example of a normal stress:

$$\tau_{xx} = \lim_{\delta A_x \to 0} \frac{\delta F_x}{\delta A_x}$$

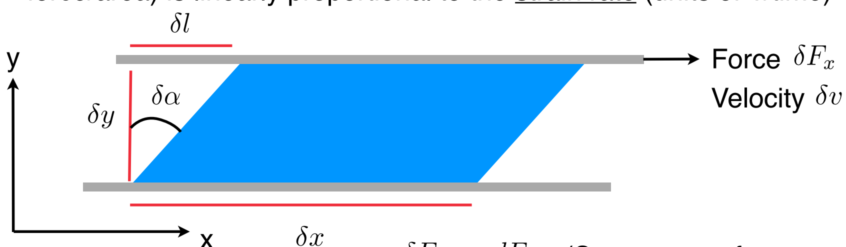
Example of a shear stress:

$$\tau_{xy} = \lim_{\delta A_x \to 0} \frac{\delta F_y}{\delta A_x}$$

Convention: first subscript indicates plane on which stress acts; second subscript indicates direction in which the stress acts.

Newtonian fluid

Definition: a Newtonian fluid is a fluid in which the stress (units of force/area) is linearly proportional to the strain rate (units of 1/time)



Shear stress: $\tau_{yx} = \lim_{\delta A_y \to 0} \frac{\delta F_x}{\delta A_y} = \frac{dF_x}{dA_y}$ (Stress on surface normal to x in the y direction)

Deformation rate: $\lim_{\delta t \to 0} \frac{\delta \alpha}{\delta t} = \frac{d\alpha}{dt}$

$$\delta l = \delta v \delta t = \delta y \delta \alpha \longrightarrow \frac{\delta \alpha}{\delta t} = \frac{\delta v}{\delta y} \longrightarrow \frac{d\alpha}{dt} = \frac{dv}{dy}$$

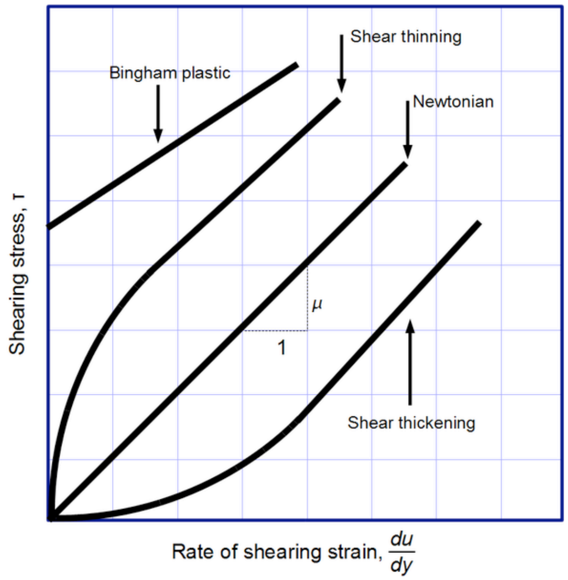
Mathematically, a Newtonian fluid satisfies $\tau_{yx} = \mu \frac{dv}{dy}$

where the proportionality constant is the viscosity of the fluid.

Types of non-Newtonian flows

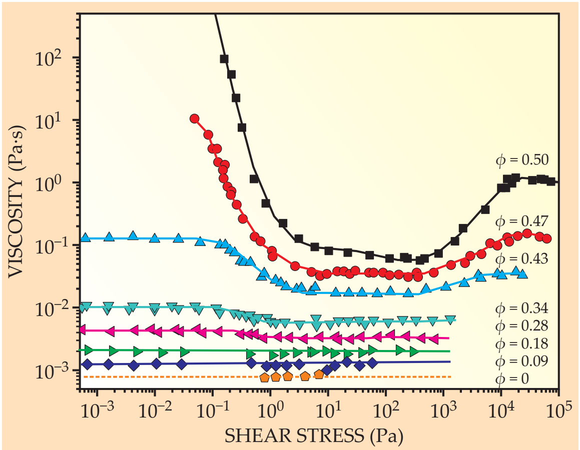
A non-Newtonian fluid is one in which the stress is **not** linearly proportional to the strain rate.

This implies that for non-Newtonian fluids, the viscosity is not constant but instead depends on the shear rate (or stress) applied.



Shear-thickening and -thinning

Definition: in a shear-thickening fluid, the viscosity is not a constant but instead increases with increasing strain rate (or with increasing stress).



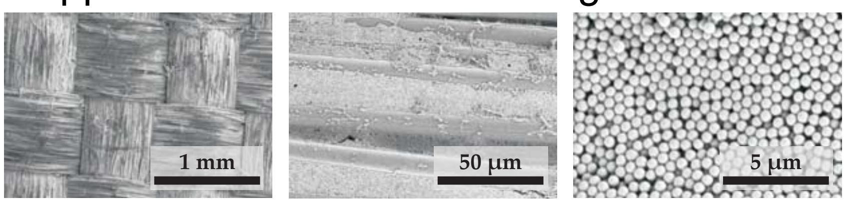
Review article: Wagner and Brady, *Phys. Today* (2009)

Example: shear-thickening fluid

Definition: in a shear-thickening fluid, the stress increases as the strain rate is increased. Recent application: pothole repair.

University of Chicago (Terry Bigioni, Eric Corwin, Matthias Mobius)

Application of shear-thickening colloids



Colloidal body armor

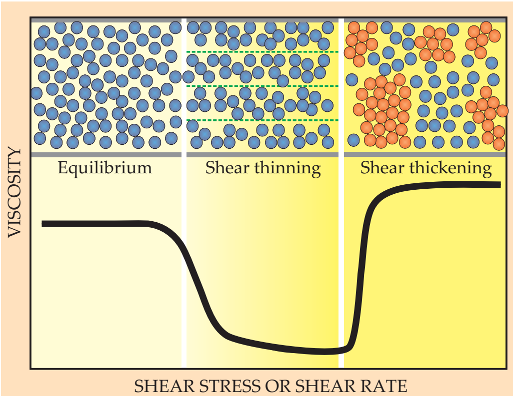
Wagner group (Delaware)

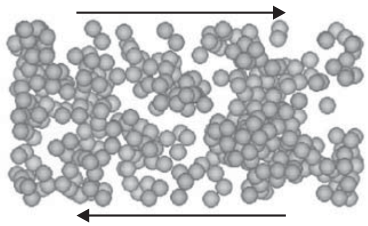
“Silly putty” for pothole repair

Case Western Reserve undergraduates

Question for physicists: microscopics?

Hypothesis: shear-thickening results from the formation of hydroclusters: clusters of particles that form when the fluid between them is expelled at high shear rates.

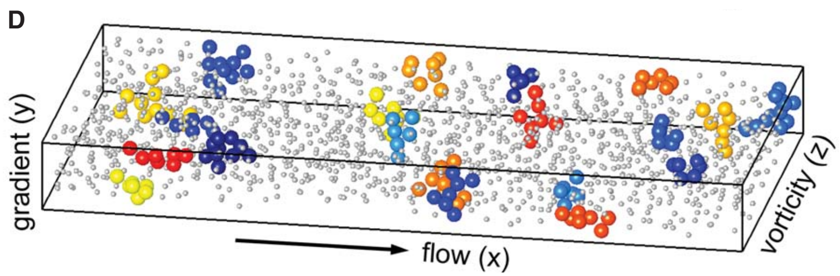




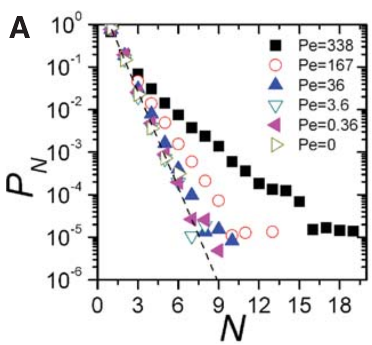
Snapshot of hydroclusters observed in simulations

Melrose and Ball, *J. Rheol.* (2004)
Review article: Wagner and Brady, *Phys. Today* (2009)

Hydroclusters in colloids



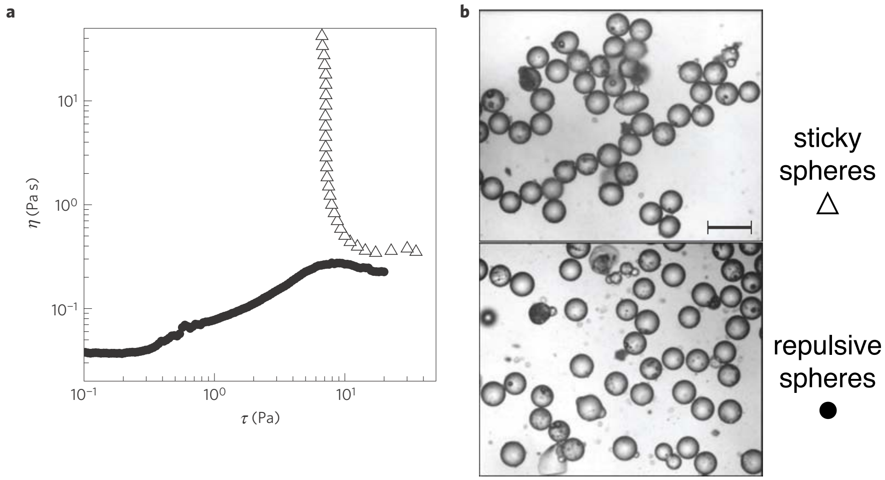
Mathematically, characterize clusters by making a histogram of the number of particles in a cluster:



Cheng *et al.*, *Science* (2011)

Shear thickening is general to suspensions

Shear thickening can be shown to exist in colloids and granular media (large particles of size >10 μm) and is suppressed when the particles are made sticky:



Brown *et al.*, *Nat. Mater.* (2010)

Summary and open questions

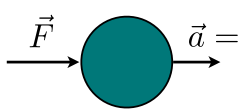
- Repulsive suspensions undergo shear-thickening at high shear rates.
- Shear thickening results from the formation of hydroclusters.
- Turning on an attraction between the particles induces permanent clusters and suppresses hydrocluster formation and shear-thickening.
- What happens with other interparticle interactions?
  - Polymer brushes (steric stabilization)
  - Patchy particles or colloidal molecules

Topic 2: low Reynolds number flows

Basic mechanics: Newton's second law

The <u>net force</u> felt by an object is proportional to its mass and is proportional to and parallel to its acceleration:

$$\sum \vec{F} = m\vec{a} = m\frac{d\vec{p}}{dt}$$



$$\vec{a} = \frac{d\vec{v}}{dt} \approx \frac{\Delta \vec{v}}{\Delta t}$$

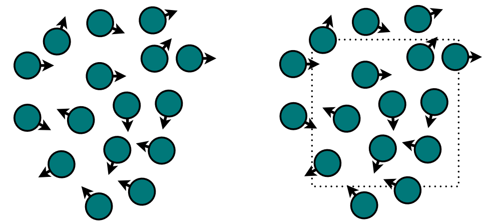
This equation applies to a <u>single</u> particle or to a single extended body.

Issue with applying Newton's law to fluids

Newton's law was developed for single particles and can be extended to small numbers of particles.

However: a flowing volume of fluid contains ≈10^{23} (Avogadro's number) of individual particles. No techniques exist for computing forces over this number of particles.

Solution: move from looking at a fixed system (number of particles) to a fixed volume (of space) through which particles move.



Two ways to have a physical property (e.g. mass, momentum) change inside the control volume:

- change inside the control volume over time
- change by flow through the control surface

Reynolds Transport Theorem

For an arbitrary extensive variable N (e.g. mass) with corresponding intensive quantity η:

$$N_{\text{sys}} = \int_{M_{\text{sys}}} \eta \, dm = \int_{V_{\text{sys}}} \eta\rho \, dV$$

N (extensive)	η (intensive)
M	1
$\mathbf{P}$	$\mathbf{v}$
$\mathbf{L}$	$\mathbf{r} \times \mathbf{v}$
E	e
S	s

REYNOLDS TRANSPORT THEOREM:

$$\boxed{\left.\frac{dN}{dt}\right|_{\text{sys}} = \frac{\partial}{\partial t}\int_{CV} \eta\rho \, dV + \int_{CS} \eta\rho \, \mathbf{v}\cdot d\mathbf{A}}$$

1 2 3

- Term 1: rate of change of any arbitrary extensive property of the system
- Term 2: time rate of change of arbitrary extensive property N, with η the corresponding intensive property
- Term 3: net rate of flux of extensive property N out through the control surface

The Navier-Stokes equations

The Navier-Stokes equations are a local form of Newton's law (conservation of momentum) expressed in control-volume form.

inertia

divergence of stress

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -\nabla p + \mu\nabla^2\mathbf{v} + \rho\mathbf{g}$$

pressure viscosity

body forces (typically gravity)

unsteady acceleration

convective acceleration

Note: on the left-hand side, the two terms represent change in the local momentum over time ("inside the control volume") and momentum flux ("through the surface")

Scaling and dimensional analysis

The Navier-Stokes equations are coupled nonlinear partial differential equations and few analytic solutions exist. Identifying the most important term for a particular problem allows the equations to be dramatically simplified.

We first identify typical length and velocity scales (that give the order of magnitude of characteristic lengths and velocities in a problem) and create new variables that are dimensionless:

Dimensionless position: $\vec{r'} = \dfrac{\vec{r}}{L}$

Dimensionless velocity: $\vec{v'} = \dfrac{\vec{v}}{V}$

Dimensionless pressure (from unit analysis): $p' = \dfrac{p}{\rho V^2}$

The Reynolds number

After all of the variables in the Navier-Stokes equations are made dimensionless, one parameter appears:

$$\rho\frac{\partial \mathbf{v}}{\partial t} + \rho\mathbf{v}\cdot\nabla\mathbf{v} = -\nabla p + \frac{1}{\mathrm{Re}}\nabla^2\mathbf{v} + \mathbf{f}$$

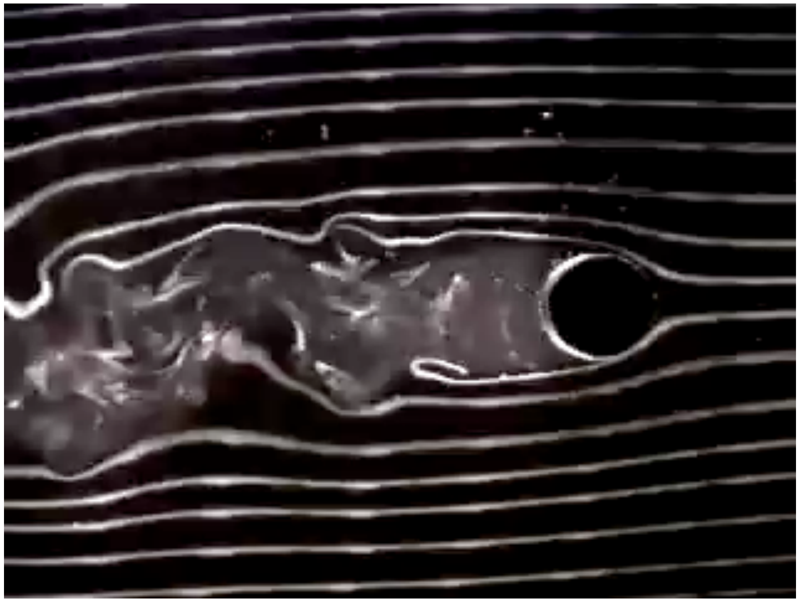
$$\mathrm{Re} = \frac{\rho V L}{\mu}$$

ρ = density of the fluid
V = typical order-of-magnitude of velocity
L = typical order-of-magnitude of length
μ = viscosity of the fluid

The Reynolds number can be represented as the ratio of two forces, the inertial force (**F** = m**a**) and the viscous force (stress area):

$$\text{Reynolds number Re} = \frac{\text{inertial force}}{\text{viscous force}} = \frac{\rho V^2 L^2}{\mu V L} = \frac{\rho V L}{\mu}$$

High Reynolds number: turbulent flow



YouTube: user guiathome

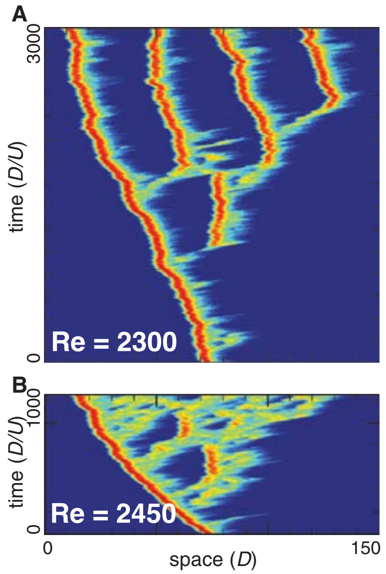
Turbulence: still an unsolved problem!

Large-scale question: what drives the formation of turbulent flow?

Old hypothesis: increase in the complexity of the time-dependence of the fluid motion.

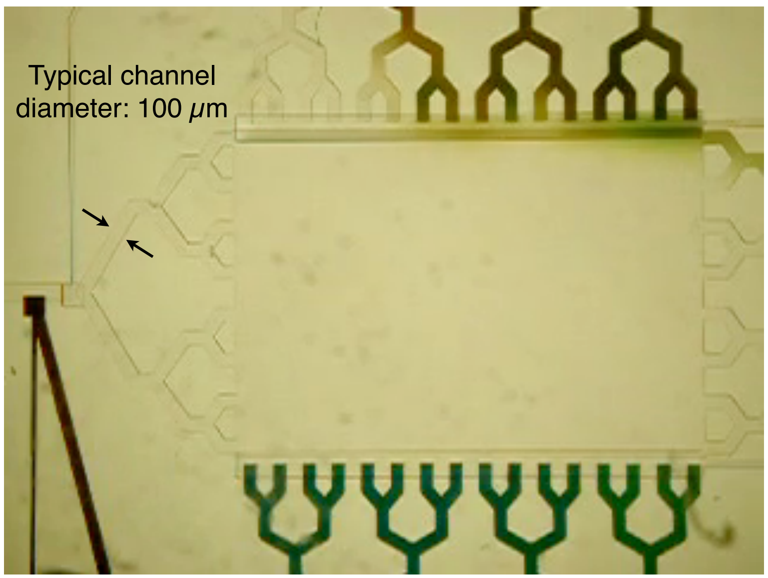
New hypothesis: spatial proliferation of domains of chaos, similar to a phase transition (as discussed in the last lecture).

Current research: study turbulence using methods from non-equilibrium phase transitions (like glasses).



Avila *et al.*, *Science* (2011)

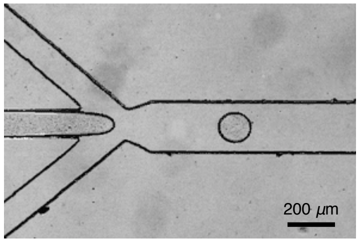
Low Reynolds number flows: microfluidics



Folch lab (University of Washington)

First example: microfluidic flows

Microfluidic flows: fluid flows in channels with typical dimensions ~100 μm



Conrad *et al.*, unpublished

Calculate typical Reynolds numbers for flow above (water drops in oil):

Typical length: $L \sim 100\,\mu\text{m}$

Typical velocity: $V \sim 1 \text{ mm/s}$

Typical density: $\rho \sim 1\,\text{g/mL}$

Typical viscosity: $\mu \sim 1\,\text{mPa} - \text{s}$

$\longrightarrow \text{Re} \sim 0.1$

Example: how fluid physics changes

In macroscale flows, turbulence helps to mix two fluids. Think of making an emulsion of oil and water by rapidly shaking the two fluids.

Question to consider: how to efficiently mix fluids in devices with micron-sized channels?

At low Re, fluid interfaces remain laminar and mixing occurs by diffusion:

$$D_0 = \frac{k_B T}{6\pi\mu a}$$

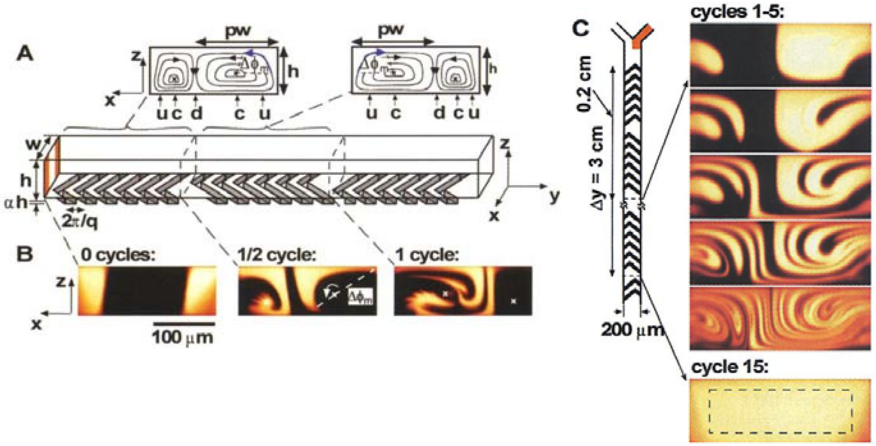
where the units of D_0 are [length^2/time].

Examples:

- D_0 for 1 μm particle: 2×10^{-13} m^2/s
  - Time to diffuse L = 100 μm: 5×10^6 s
- D_0 for 1 *n*m particle: 2×10^{-10} m^2/s
  - Time to diffuse L = 100 μm: 5×10^3 s

Mixing via designed turbulence

Strategy: fabricate micromixers within a microfluidic device that stretch and fold the fluid and enhance mixing, similar to how turbulence mixes a fluid:



Stroock *et al.*, *Science* (2002)

Mixing and folding of fluid elements

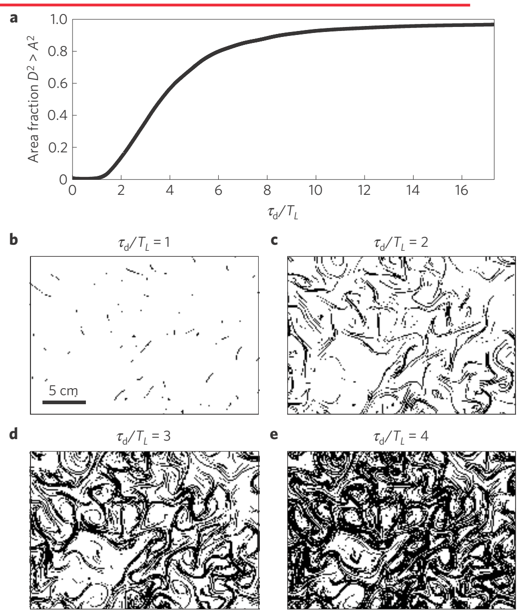
Stretching: short-time linear affine deformation

- Composed of rotation, shear, dilation, compression

Folding: long-time nonlinear non-affine deformation

- Requires irreversible rearrangements in the fluid

By mathematical analysis of the ratio of affine to non-affine transformation, can show that folding (non-affine) deformations occur only after elongation (i.e. at long times).



Kelley and Ouellette, *Nat. Phys.* (2011)

Summary and open questions

- Turbulence is still an unsolved problem, but shares features with non-equilibrium phase transitions.
- Flows in microfluidic devices are at low Reynolds number, and flows are typically laminar (not turbulent).
- Mixing therefore requires special structures to irreversibly deform the fluid on long times.
  - What engineered structures create fastest mixing?